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A Heuristic Approach for the Independent Set Problem

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Abstract:

Given a graph G =(V, E), the independent set problem is to find a maximum cardinality subset S of V such that all vertices in S are pairwise non-adjacent. This problem is NP-hard in general but can be solved in polynomial time on some special graph classes, like bipartite graphs, chair-free graphs or claw-free graphs. In this article, we propose a heuristic algorithm for this problem. The results of its application on some famous graphs taken from the literature, with known optimal solutions demonstrates that this approach is effective.

INTRODUCTION

In graph theory, the stable set (or maximum independent set) of a graph is a maximum cardinality subset of vertices in which no two vertices are adjacent. This problem which has many applications in computer science and operations research is computationally intractable in general [5] but can be solved in polynomial time on some special graph classes, like bipartite graphs [8], chair-free graphs [3] or claw-free graphs [7] [11]. Therefore, for large and hard instances one must design heuristic approaches to obtain near optimal solutions within reasonable time.

Given an undirected graph $G = (V, E)$ with vertex set $V = V(G)$ of cardinality $|V(G)| = n$, and edge set $E = E(G)$ of cardinality $|E(G)| = m$.

The neighborhood of a vertex $v \in V$ is the set $N(v) = \{u \in V : vu \in E\}$.

A set $S \subseteq V(G)$ is independent if no two vertices from S are adjacent; by Ind(G) we mean the set of all the independent sets of G. An independent set of maximum size will be referred to a maximum stable set of G.

A vertex cover is a subset $Vc \subseteq V$, such that every edge in G has at least one endpoint in Vc.

The rest of this paper is organized as follows: In section 2 we describe the proposed approach, whereas section 3 provides some examples of the application of the proposed heuristic on some famous graphs taken from the literature. Finally, concluding remarks are given in section 4.

PROPOSED APPROACH

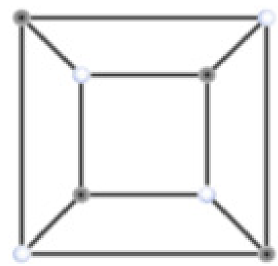
The main idea of the proposed heuristic is to build a maximal stable set I by considering incrementally a minimal vertex cover Vc.

The pseudo-code of the proposed heuristic algorithm is given below.

```
Input: G(V, E)
Output: A stable set I of G.
J← [1,n]
I← []
for i=1:n
        J1 ← {j∈ J/ a(j,1)=i}
        J2 ← {j∈ J/ a(j,2)=i}
        c←{a(J1,1)} ∪{a(J1,2)}
        v(i)←c
        L(i)←|c|
end for
i1←ArgMin(L(i))
        i∈J
Vc←v(i1)
I(1)←{i1}
K ←J-(I ∪ Vc)
        while K≠∅ do
                w← []
                L← []
                for i=1:n
                        w(i)← v(i)∪Vc
                        if i∈K
                                L(i)←|w(i)|
                        else
                                L(i)←n+1
                        end
                end for
                i1←ArgMin(L(i))
                        i∈J
                I←I ∪ {i1}
                Vc←w(i1)
                K ←J-(I ∪ Vc)
        end while
output  I
```

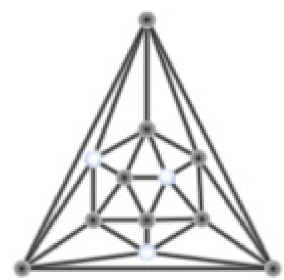
3. Some examples

In the following examples, we demonstrates the application of the proposed heuristic on some instance of famous graphs defined by their adjacency matrix. We report, the outputs of the heuristic as a list of the maximal stable sets S_v that contain vertex v (v=1,...,n), the independence number α of G and the stable set S of G.



(The vertices of S=I are colored in white in the following figures)

fig.1 The graph of the Cube



α=4 S={ 1 , 3 , 5 , 7 }

fig.2 the Icosahedron graph [10]

α=3 S= { 1 , 4 , 11 }

Fig. 1 A sample line graph using colors which contrast well both on screen and on a black-and-white hardcopy

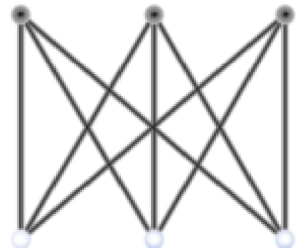


fig.3 The graph K3,3 [6]
α=3 S= { 1 , 2 , 3 }

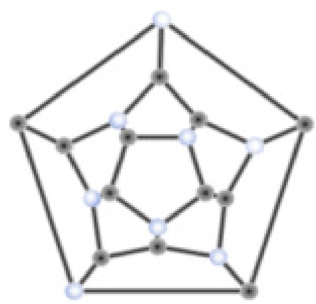


fig.4 The Dodecahedron graph [10]
α=8 S={ 1 , 3 , 6 , 8 , 11 , 13 , 16 , 18 }

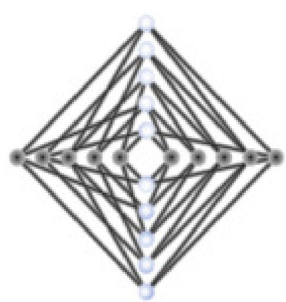


fig.5 The Folkman graph [4]
α=10 S= { 1 , 10 , 2 , 9 , 3 , 8 , 4 , 5 , 6 , 7 }

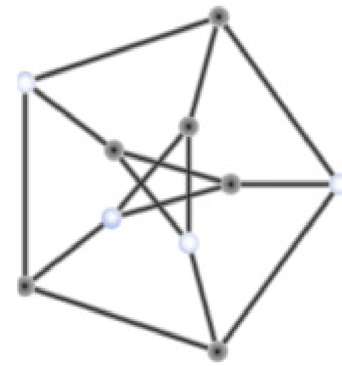


fig.6 The Petersen graph [9]
α=4 S={ 1 , 3 , 6 , 10 }

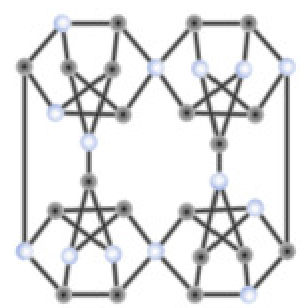


fig.7 The Thomassen graph [12]

α=14 S= { 1 , 2 , 15 , 8 , 13 , 9 , 17 , 19 , 29 , 25 , 21 , 32 , 24 , 33 }

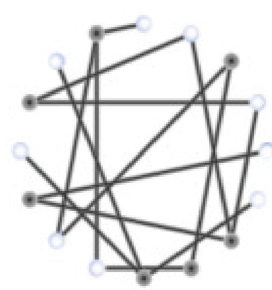


fig.9 The Bondy-Murty graph G4 [2]
α=9 S={ 1 , 3 , 4 , 5 , 12 , 14 , 16 , 9 , 2 }

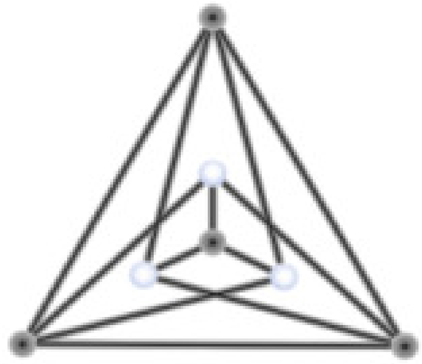


fig.8 The Bondy-Murty graph G1 [2]
α=3 S={ 4 , 5 , 6 }

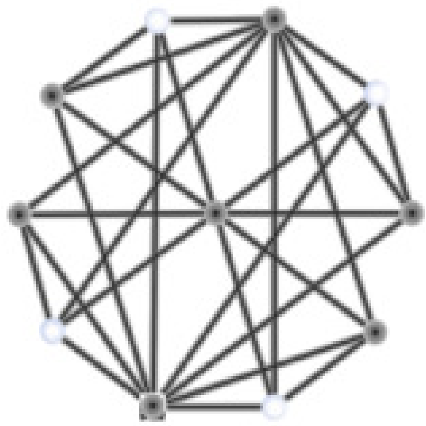


fig.9 The Bondy-Murty graph G2 [2]
α=4 S={3 , 5 , 8,10 }

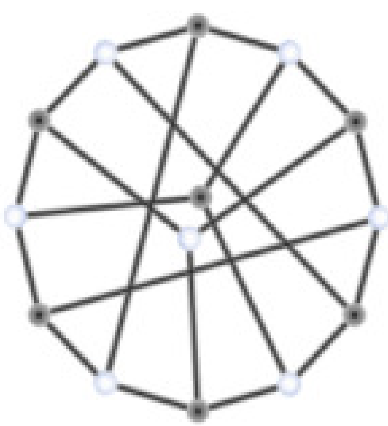


fig.10:The Bondy-Murty graph G3 [2]
α=7 S={ 1,3 ,7 ,13 ,5 , 9 ,11 }

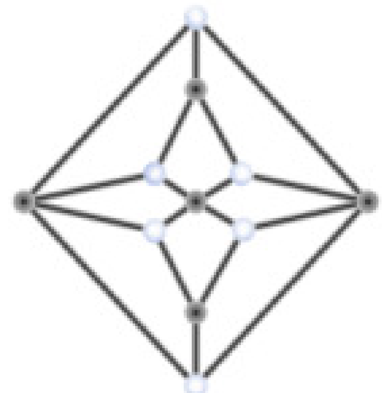


fig.11:The Grötzsch graph [15]
α=5 S={ 2, 3 , 4 , 5 , 6}

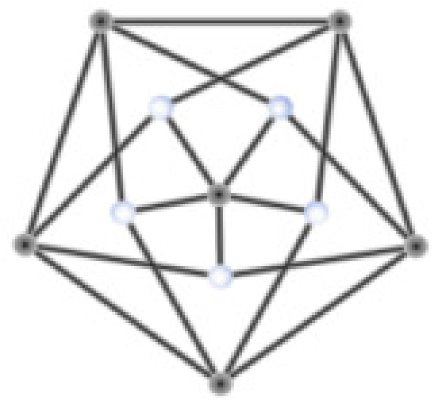


fig.12:The Herschel graph [16]
α=6 S={2 , 4 , 9, 5 , 7 , 11}

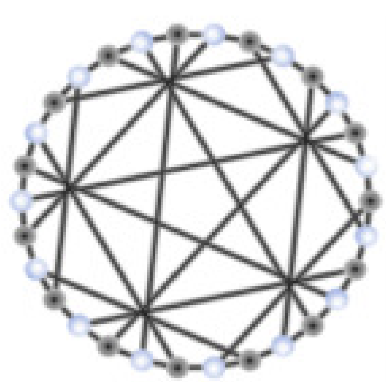
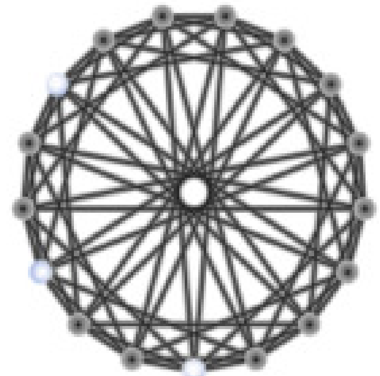


fig.13:The Tutte-Coxeter graph [17]

α=15 S={1 3 5 17 23 25 7 9 11 29 13 15 19 21 27}

FIG.14:THE RAMSEY GRAPH R(4,4) [14]

α=3 S={1 , 4 , 7}



CONCLUSION

In the above, we described a heuristic algorithm for tackling the problem of finding a maximum independent set of a given graph. We reported the results of the applications of this heuristic on some famous graphs with known maximum stable set, and demonstrates the effectiveness of this approach.

Further research will be concerned with the improvement of this heuristic by developing a parallel version in order to speed up its performances on large graph instances. In a forthcoming work, we will also try to identify the class of graphs for which this heuristic always finds the optimal solutions.

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